Family Name	
Given Names	
Student Number	
Teaching Period	Semester 1, 2016

FINAL EXAMINATION	DURATION
ENG480 – Applied Fluid Mechanics	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

This examination consists of **6 (six) questions**. Candidates are only required to answer **ANY 4 (four) questions**.

The total mark for this examination is **40 marks**.

EXAM CONDITIONS

<u>You may begin writing from the commencement of the examination session.</u> The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 16 Page Book Formula Sheet/s Reference Information

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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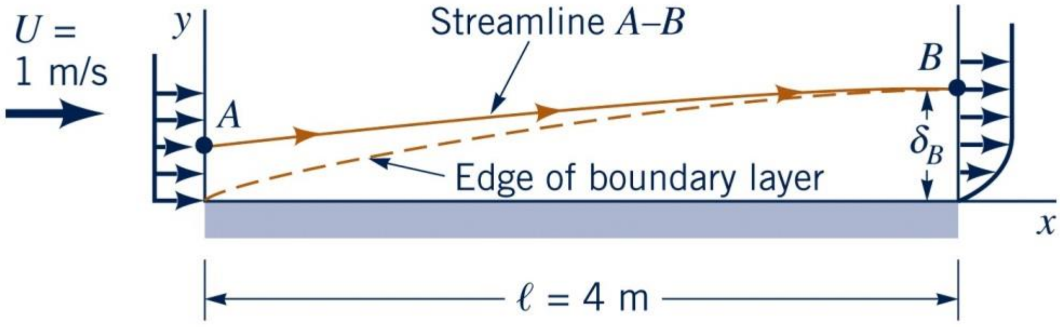
Question 1

Because of the velocity deficit, U-u, in the boundary layer, the streamlines for flow past a flat plate are not exactly parallel to the plate. This deviation can be determined by use of displacement thickness, δ^*. For air blowing past the flat plate shown in the figure, plot the streamline A-B that passes through the edge of the boundary later ($y=\delta_b$ @ $x = l$) at point B. That is, plot $y = y(x)$ for streamline A-B. Assume laminar boundary layer flow.

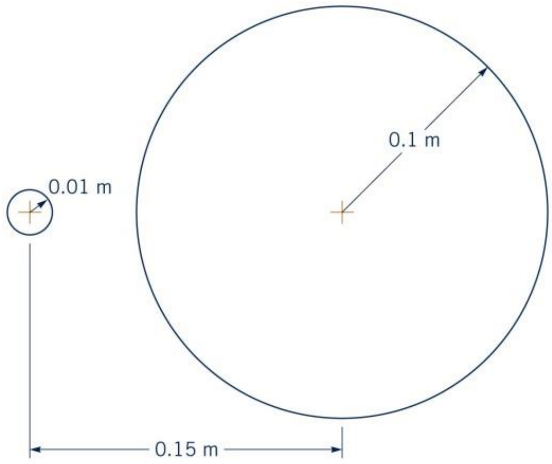
(10 marks)



Question 2

(a) At a given instant, two pressure waves, each moving at the speed of sound, emitted by a point sources moving with constant velocity in a fluid at rest is shown. Determine the Mach number involved, and indicate with a sketch the instantaneous location of the point source.

(5 marks)



(b) At the seashore, you observe a high-speed aircraft moving overhead at an elevation of 3048 m. you hear the plane 8 s after it passes directly overhead. Using a nominal air temperature of 4.4 °C, estimate the Mach number and speed of the aircraft.

(5 marks)

Question 3

(a) For an isentropic flow with constants specific heats C_v and C_p, derive the following relationships:

$$\left[\frac{T_2}{T_1}\right]^{\frac{k}{k-1}} = \left[\frac{\rho_2}{\rho_1}\right]^{k} = \left[\frac{p_2}{p_1}\right]$$

(5 marks)

(b) For a Fanno flow, prove/derive the following equation, and in doing so, show that when the flow is subsonic, friction accelerates the fluid, and when the flow is supersonic, friction decelerates the fluid.

$$\frac{dV}{V} = \frac{fk(\frac{Ma^2}{2})(\frac{dx}{D})}{1 - Ma^2}$$

(5 marks)

Question 4

An ideal gas enters a frictionless constant area duct with the following properties: T_0 = 293 K, P_0 = 101 kPa(abs) and Ma_1 = 0.2. For a Rayleigh flow, determine corresponding values of fluid temperature and entropy change for various levels of pressure and plot the Rayleigh line if the gas is helium.

(10 marks)

Question 5

Just upstream of a normal shock in an airflow, Ma = 3.0, T = 333 K and p = 206 kPa. Estimate the values of Ma, T_0, T, p_0, p and V downstream of the shock.

(10 marks)

Question 6

A normal shock is positioned in the diverging portion of a frictionless, adiabatic, converging-diverging airflow duct where the cross-sectional area is 0.009 m^2 and the local Mach number is 2.0. Upstream of the shock, p_0 = 1378 kPa and T_0 = 667 K. If the duct exit area is 0.014 m^2, determine the exit area temperature and pressure and the duct mass flowrate.

(10 marks)

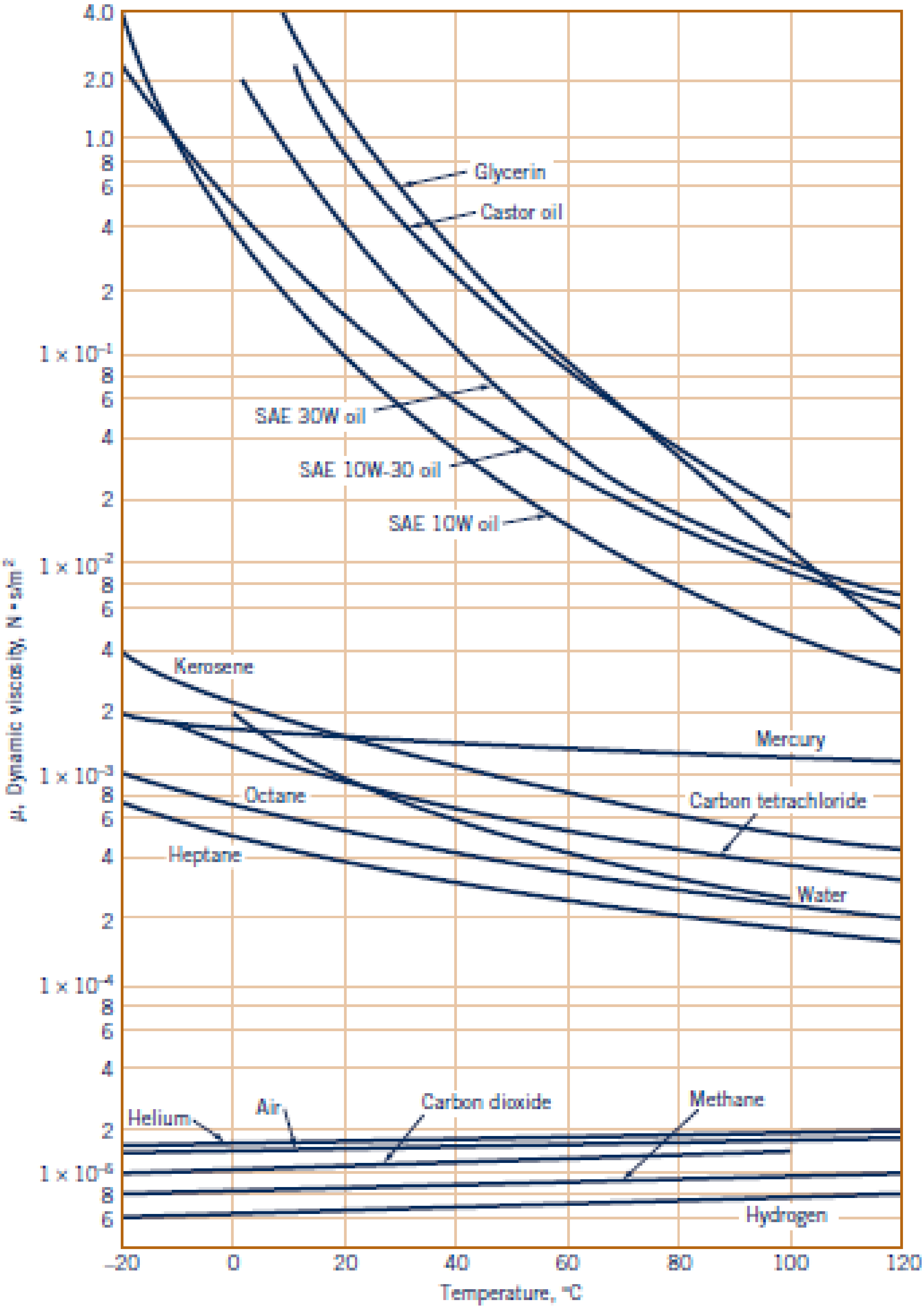
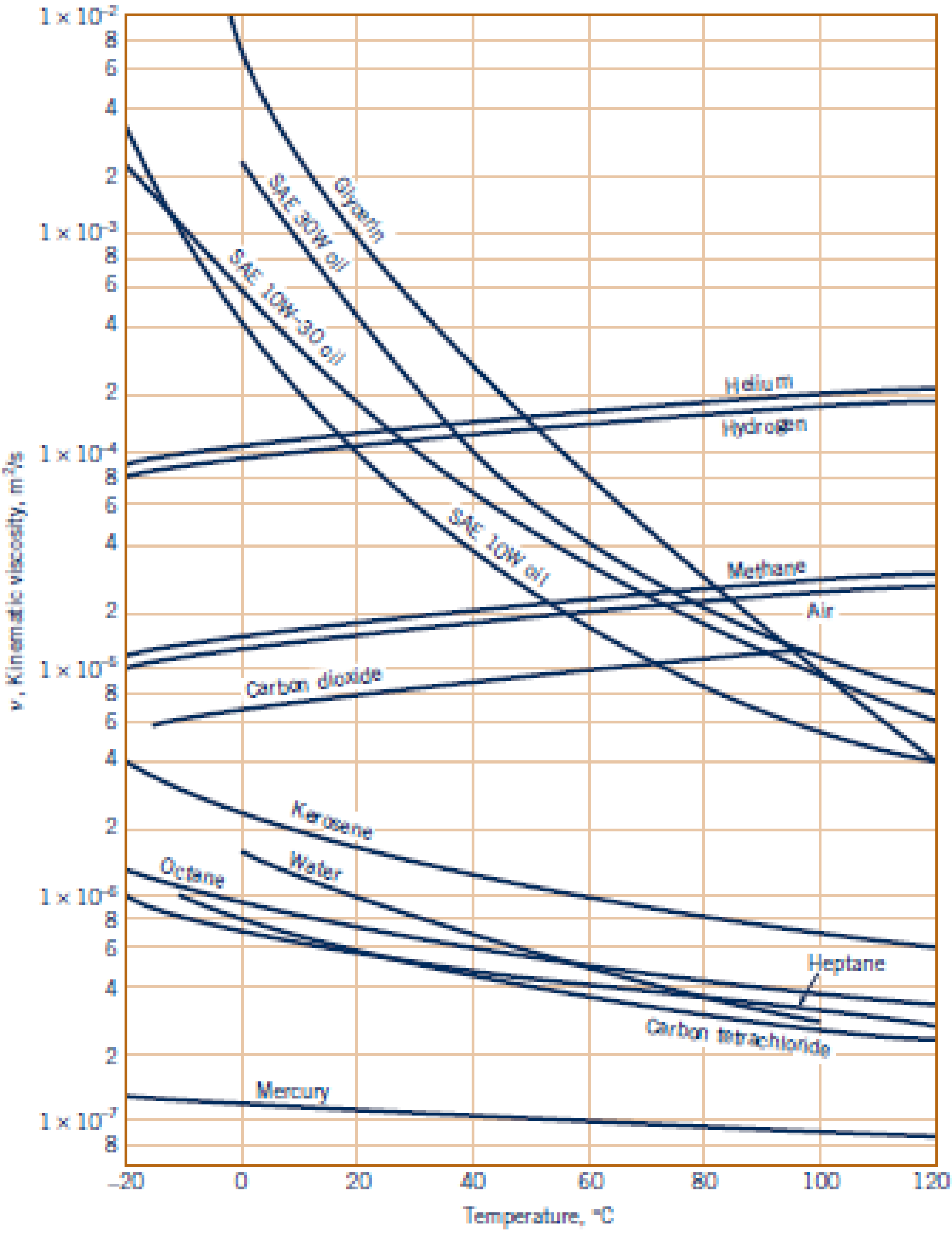


Figure B.1 **Dynamic (absolute) viscosity of common fluids as a function of temperature. To convert to BG units of lb · s/ft² multiply N · s/m² by 2.089 × 10⁻². (Curves from R. W. Fox and A. T. McDonald, *Introduction to Fluid Mechanics*, 3rd Ed., Wiley, New York, 1985. Used by permission.)**



■ **Figure B.2** Kinematic viscosity of common fluids (at atmospheric pressure) as a function of temperature. To convert to BG units of ft^2/s multiply m^2/s by 10.76. (Curves from R. W. Fox and A. T. McDonald, *Introduction to Fluid Mechanics*, 3rd Ed., Wiley, New York, 1985. Used by permission.)

Table B.2
Physical Properties of Water (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight[b], γ (kN/m^3)	Dynamic Viscosity, μ (N·s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension[c], σ (N/m)	Vapor Pressure, p_v [N/m^2(abs)]	Speed of Sound[d], c (m/s)
0	999.9	9.806	1.787 E − 3	1.787 E − 6	7.56 E − 2	6.105 E + 2	1403
5	1000.0	9.807	1.519 E − 3	1.519 E − 6	7.49 E − 2	8.722 E + 2	1427
10	999.7	9.804	1.307 E − 3	1.307 E − 6	7.42 E − 2	1.228 E + 3	1447
20	998.2	9.789	1.002 E − 3	1.004 E − 6	7.28 E − 2	2.338 E + 3	1481
30	995.7	9.765	7.975 E − 4	8.009 E − 7	7.12 E − 2	4.243 E + 3	1507
40	992.2	9.731	6.529 E − 4	6.580 E − 7	6.96 E − 2	7.376 E + 3	1526
50	988.1	9.690	5.468 E − 4	5.534 E − 7	6.79 E − 2	1.233 E + 4	1541
60	983.2	9.642	4.665 E − 4	4.745 E − 7	6.62 E − 2	1.992 E + 4	1552
70	977.8	9.589	4.042 E − 4	4.134 E − 7	6.44 E − 2	3.116 E + 4	1555
80	971.8	9.530	3.547 E − 4	3.650 E − 7	6.26 E − 2	4.734 E + 4	1555
90	965.3	9.467	3.147 E − 4	3.260 E − 7	6.08 E − 2	7.010 E + 4	1550
100	958.4	9.399	2.818 E − 4	2.940 E − 7	5.89 E − 2	1.013 E + 5	1543

[a]Based on data from *Handbook of Chemistry and Physics*, 69th Ed., CRC Press, 1988.
[b]Density and specific weight are related through the equation $\gamma = \rho g$. For this table, $g = 9.807$ m/s^2.
[c]In contact with air.
[d]Based on data from R. D. Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Co., Inc., New York, 1984.

Table B.4
Physical Properties of Air at Standard Atmospheric Pressure (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight[b], γ (N/m^3)	Dynamic Viscosity, μ (N·s/m^2)	Kinematic Viscosity, ν (m^2/s)	Specific Heat Ratio, k (—)	Speed of Sound, c (m/s)
−40	1.514	14.85	1.57 E − 5	1.04 E − 5	1.401	306.2
−20	1.395	13.68	1.63 E − 5	1.17 E − 5	1.401	319.1
0	1.292	12.67	1.71 E − 5	1.32 E − 5	1.401	331.4
5	1.269	12.45	1.73 E − 5	1.36 E − 5	1.401	334.4
10	1.247	12.23	1.76 E − 5	1.41 E − 5	1.401	337.4
15	1.225	12.01	1.80 E − 5	1.47 E − 5	1.401	340.4
20	1.204	11.81	1.82 E − 5	1.51 E − 5	1.401	343.3
25	1.184	11.61	1.85 E − 5	1.56 E − 5	1.401	346.3
30	1.165	11.43	1.86 E − 5	1.60 E − 5	1.400	349.1
40	1.127	11.05	1.87 E − 5	1.66 E − 5	1.400	354.7
50	1.109	10.88	1.95 E − 5	1.76 E − 5	1.400	360.3
60	1.060	10.40	1.97 E − 5	1.86 E − 5	1.399	365.7
70	1.029	10.09	2.03 E − 5	1.97 E − 5	1.399	371.2
80	0.9996	9.803	2.07 E − 5	2.07 E − 5	1.399	376.6
90	0.9721	9.533	2.14 E − 5	2.20 E − 5	1.398	381.7
100	0.9461	9.278	2.17 E − 5	2.29 E − 5	1.397	386.9
200	0.7461	7.317	2.53 E − 5	3.39 E − 5	1.390	434.5
300	0.6159	6.040	2.98 E − 5	4.84 E − 5	1.379	476.3
400	0.5243	5.142	3.32 E − 5	6.34 E − 5	1.368	514.1
500	0.4565	4.477	3.64 E − 5	7.97 E − 5	1.357	548.8
1000	0.2772	2.719	5.04 E − 5	1.82 E − 4	1.321	694.8

[a]Based on data from R. D. Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Co., Inc., New York, 1984.
[b]Density and specific weight are related through the equation $\gamma = \rho g$. For this table $g = 9.807$ m/s^2.

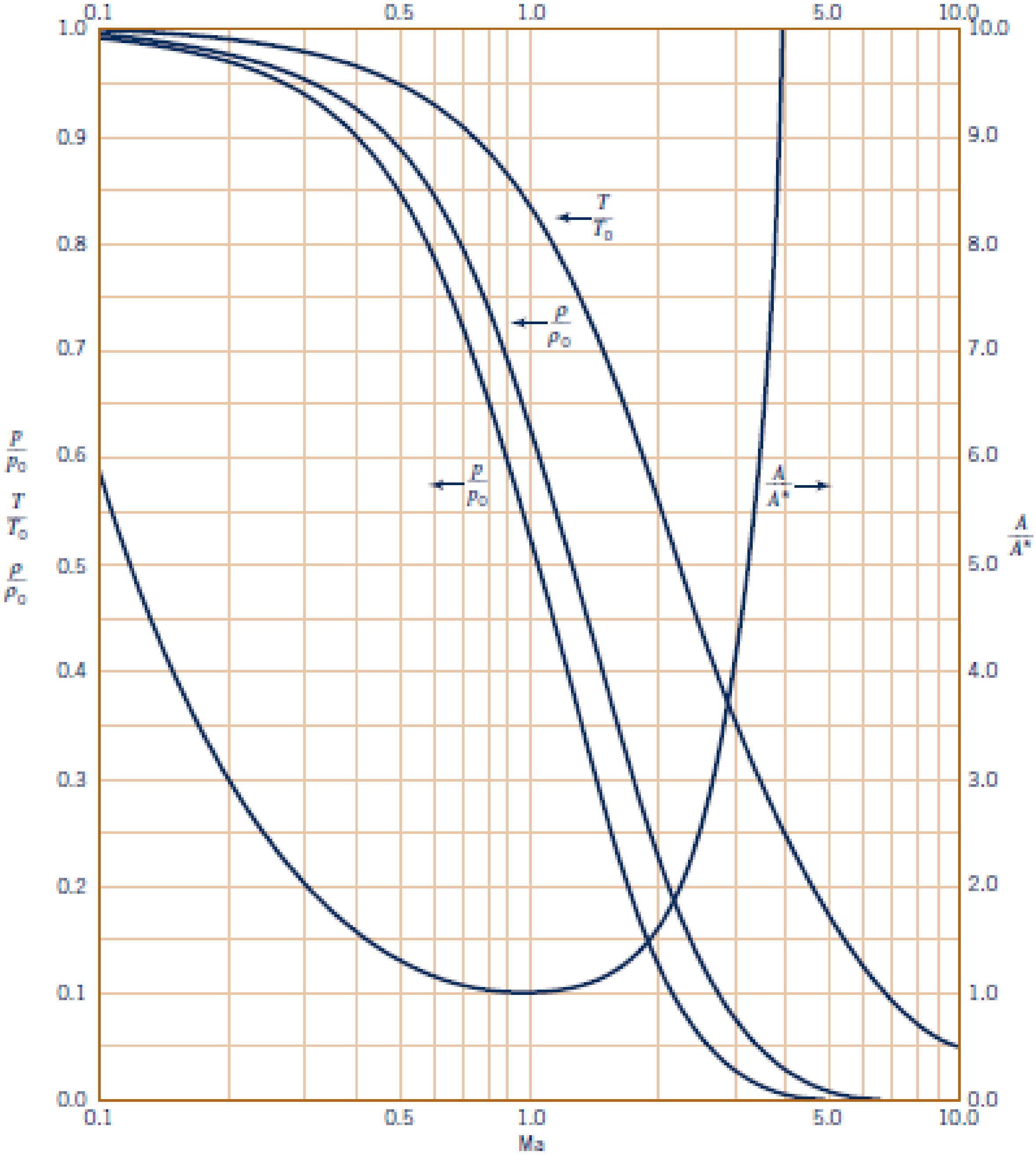
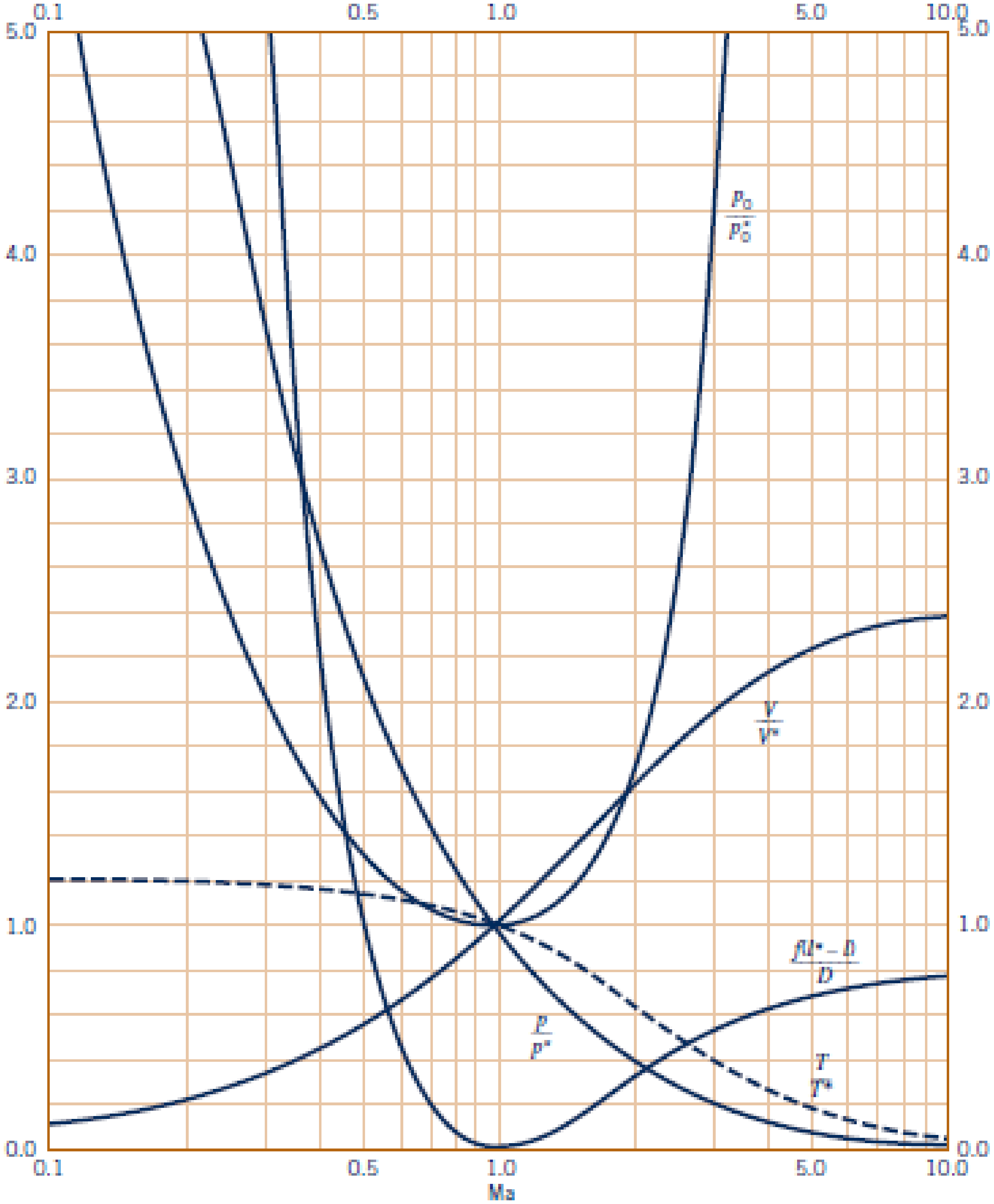


Figure D.1 Isentropic flow of an ideal gas with $k = 1.4$. (Graph provided by Dr. Bruce A. Reichert.)



■ Figure D.2 Fanno flow of an ideal gas with $k = 1.4$. (Graph provided by Dr. Bruce A. Reichert.)

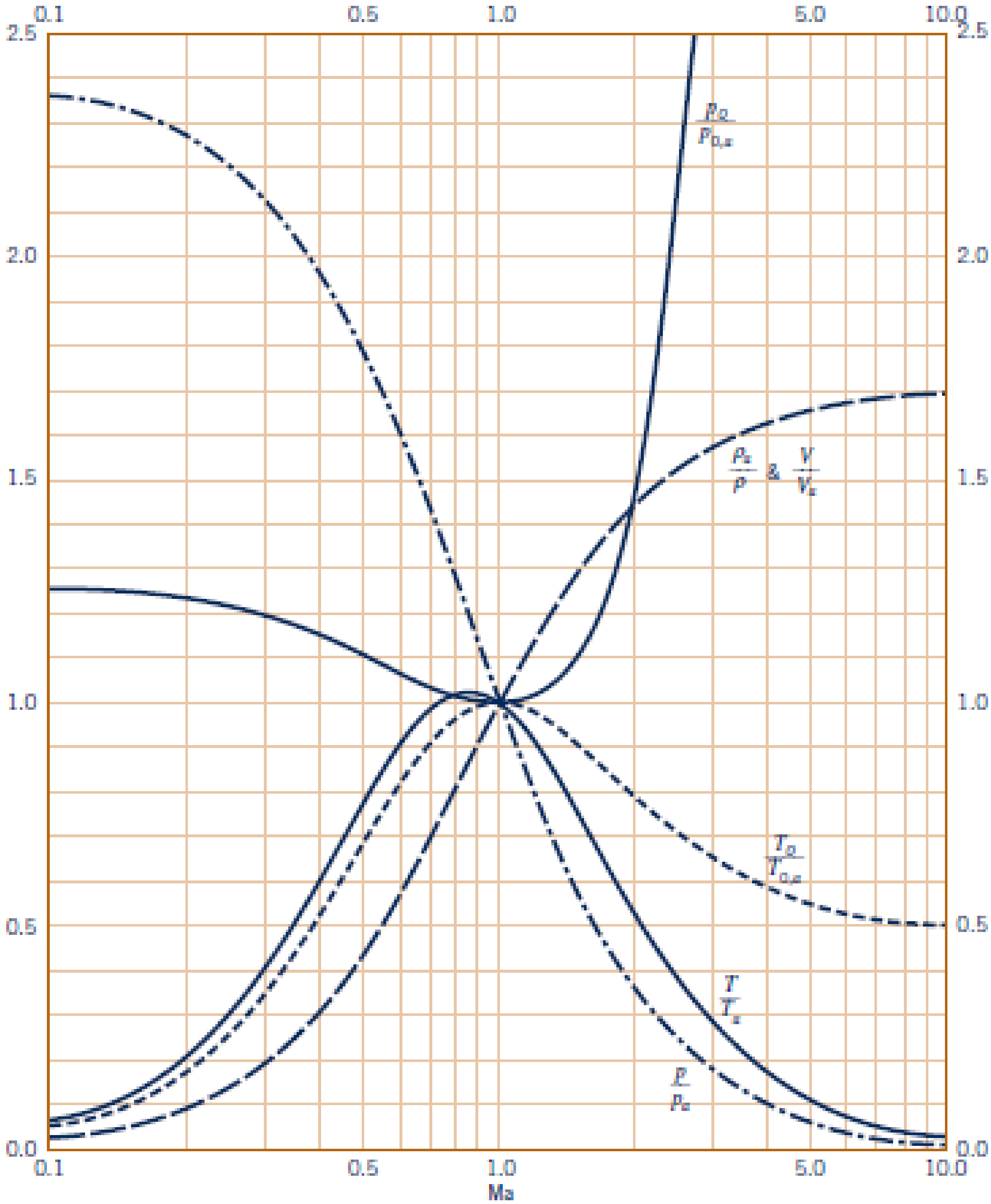


Figure D.3 Rayleigh flow of an ideal gas with $k = 1.4$. (Graph provided by Dr. Bruce A. Reichert.)

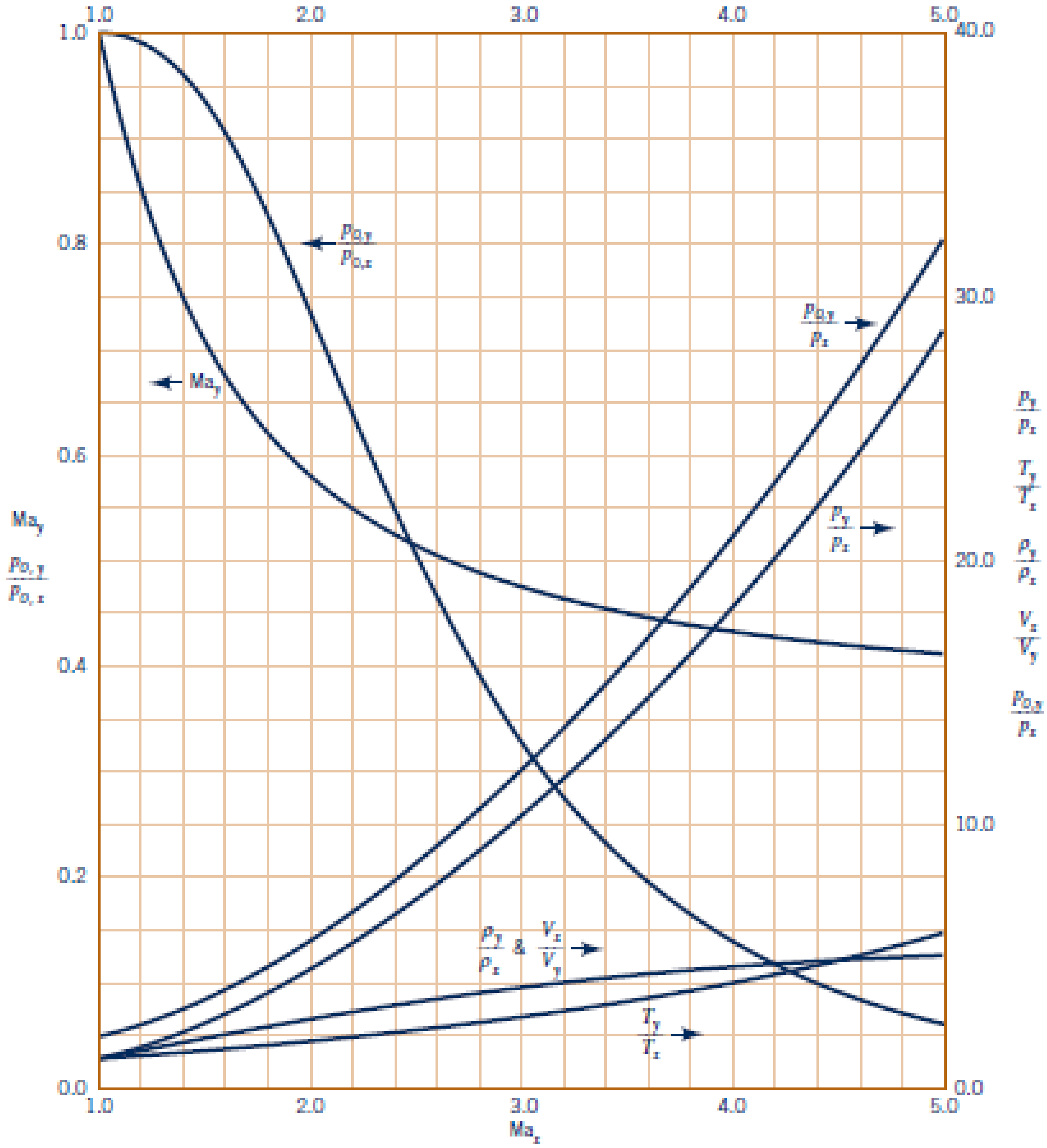


Figure D.4 Normal shock flow of an ideal gas with $k = 1.4$. (Graph provided by Dr. Bruce A. Reichert.)

■ TABLE 1.6

Approximate Physical Properties of Some Common Liquids (SI Units)

Liquid	Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight, γ (kN/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension,[a] σ (N/m)	Vapor Pressure, p_v [N/m^2 (abs)]	Bulk Modulus,[b] E_v (N/m^2)
Carbon tetrachloride	20	1,590	15.6	9.58 E − 4	6.03 E − 7	2.69 E − 2	1.3 E + 4	1.31 E + 9
Ethyl alcohol	20	789	7.74	1.19 E − 3	1.51 E − 6	2.28 E − 2	5.9 E + 3	1.06 E + 9
Gasoline[c]	15.6	680	6.67	3.1 E − 4	4.6 E − 7	2.2 E − 2	5.5 E + 4	1.3 E + 9
Glycerin	20	1,260	12.4	1.50 E + 0	1.19 E − 3	6.33 E − 2	1.4 E − 2	4.52 E + 9
Mercury	20	13,600	133	1.57 E − 3	1.15 E − 7	4.66 E − 1	1.6 E − 1	2.85 E + 10
SAE 30 oil[c]	15.6	912	8.95	3.8 E − 1	4.2 E − 4	3.6 E − 2	—	1.5 E + 9
Seawater	15.6	1,030	10.1	1.20 E − 3	1.17 E − 6	7.34 E − 2	1.77 E + 3	2.34 E + 9
Water	15.6	999	9.80	1.12 E − 3	1.12 E − 6	7.34 E − 2	1.77 E + 3	2.15 E + 9

[a] In contact with air.
[b] Isentropic bulk modulus calculated from speed of sound.
[c] Typical values. Properties of petroleum products vary.

■ TABLE 1.8

Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (SI Units)

Gas	Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight, γ (N/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Gas Constant,[a] R (J/kg · K)	Specific Heat Ratio,[b] k
Air (standard)	15	1.23 E + 0	1.20 E + 1	1.79 E − 5	1.46 E − 5	2.869 E + 2	1.40
Carbon dioxide	20	1.83 E + 0	1.80 E + 1	1.47 E − 5	8.03 E − 6	1.889 E + 2	1.30
Helium	20	1.66 E − 1	1.63 E + 0	1.94 E − 5	1.15 E − 4	2.077 E + 3	1.66
Hydrogen	20	8.38 E − 2	8.22 E − 1	8.84 E − 6	1.05 E − 4	4.124 E + 3	1.41
Methane (natural gas)	20	6.67 E − 1	6.54 E + 0	1.10 E − 5	1.65 E − 5	5.183 E + 2	1.31
Nitrogen	20	1.16 E + 0	1.14 E + 1	1.76 E − 5	1.52 E − 5	2.968 E + 2	1.40
Oxygen	20	1.33 E + 0	1.30 E + 1	2.04 E − 5	1.53 E − 5	2.598 E + 2	1.40

[a] Values of the gas constant are independent of temperature.
[b] Values of the specific heat ratio depend only slightly on temperature.

Table E.1
Listing by Physical Quantity

To convert from	to	Multiply by
Acceleration		
foot/second^2	meter/second^2	3.048 E − 1*
free fall, standard	meter/second^2	9.806 65 E + 0*
gal (galileo)	meter/second^2	1.00 E − 2*
inch/second^2	meter/second^2	2.54 E − 2*
Area		
acre	meter^2	4.046 856 422 4 E + 3*
are	meter^2	1.00 E + 2*
barn	meter^2	1.00 E − 28*
foot^2	meter^2	9.290 304 E − 2*
hectare	meter^2	1.00 E + 4*
inch^2	meter^2	6.4516 E − 4*
mile^2 (U.S. statute)	meter2	2.589 988 110 336 E + 6*
section	meter^2	2.589 988 110 336 E + 6*
township	meter2	9.323 957 2 E + 7
yard^2	meter^2	8.361 273 6 E − 1*
Density		
gram/centimeter^3	kilogram/meter^3	1.00 E + 3*
lbm/inch^3	kilogram/meter^3	2.767 990 5 E + 4
lbm/foot^3	kilogram/meter^3	1.601 846 3 E + 1
slug/foot^3	kilogram/meter^3	5.153 79 E + 2

Table E.1 (continued)

To convert from	to	Multiply by
Energy		
British thermal unit: (IST after 1956)	joule	1.055 056 E + 3
British thermal unit (thermochemical)	joule	1.054 350 E + 3
calorie (International Steam Table)	joule	4.1868 E + 0
calorie (thermochemical)	joule	4.184 E + 0*
calorie (kilogram, International Steam Table)	joule	4.1868 E + 3
calorie (kilogram, thermochemical)	joule	4.184 E + 3*
electron volt	joule	1.602 191 7 E − 19
erg	joule	1.00 E − 7*
foot lbf	joule	1.355 817 9 E + 0
foot poundal	joule	4.214 011 0 E − 2
joule (international of 1948)	joule	1.000 165 E + 0
kilocalorie (Internation Steam Table)	joule	4.1868 E + 3
kilocalorie (thermochemical)	joule	4.184 E + 3*
kilowatt hour	joule	3.60 E + 6*
watt hour	joule	3.60 E + 3*
Force		
dyne	newton	1.00 E − 5*
kilogram force (kgf)	newton	9.806 65 E + 0*
kilopound force	newton	9.806 65 E + 0*
kip	newton	4.448 221 615 260 5 E + 3*
lbf (pound force, avoirdupois)	newton	4.448 221 615 260 5 E + 0*
ounce force (avoirdupois)	newton	2.780 138 5 E − 1
pound force, lbf (avoirdupois)	newton	4.448 221 615 260 5 E + 0*
poundal	newton	1.382 549 543 76 E − 1*
Length		
angstrom	meter	1.00 E − 10*
astronomical unit (IAU)	meter	1.496 00 E + 11
cubit	meter	4.572 E − 1*
fathom	meter	1.8288 E + 0*
foot	meter	3.048 E − 1*
furlong	meter	2.011 68 E + 2*
hand	meter	1.016 E − 1*
inch	meter	2.54 E − 2*
league (international nautical)	meter	5.556 E + 3*
light year	meter	9.460 55 E + 15
meter	wavelengths Kr 86	1.650 763 73 E + 6*
micron	meter	1.00 E − 6*
mil	meter	2.54 E − 5*
mile (U.S. statute)	meter	1.609 344 E + 3*
nautical mile (U.S.)	meter	1.852 E + 3*
rod	meter	5.0292 E + 0*
yard	meter	9.144 E − 1*

Table E.1 (continued)

To convert from	to	Multiply by
Mass		
carat (metric)	kilogram	2.00 E − 4*
grain	kilogram	6.479 891 E − 5*
gram	kilogram	1.00 E − 3*
ounce mass (avoirdupois)	kilogram	2.834 952 312 5 E − 2*
pound mass, lbm (avoirdupois)	kilogram	4.535 923 7 E − 1*
slug	kilogram	1.459 390 29 E + 1
ton (long)	kilogram	1.016 046 908 8 E + 3*
ton (metric)	kilogram	1.00 E + 3*
ton (short, 2000 pound)	kilogram	9.071 847 4 E + 2*
tonne	kilogram	1.00 E + 3*
Power		
Btu (thermochemical)/second	watt	1.054 350 264 488 E + 3
calorie (thermochemical)/second	watt	4.184 E + 0*
foot lbf/second	watt	1.355 817 9 E + 0
horsepower (550 foot lbf/second)	watt	7.456 998 7 E + 2
kilocalorie (thermochemical)/second	watt	4.184 E + 3*
watt (international of 1948)	watt	1.000 165 E + 0
Pressure		
atmosphere	newton/meter^2	1.013 25 E + 5*
bar	newton/meter^2	1.00 E + 5*
barye	newton/meter^2	1.00 E − 1*
centimeter of mercury (0 °C)	newton/meter^2	1.333 22 E + 3
centimeter of water (4 °C)	newton/meter^2	9.806 38 E + 1
dyne/centimeter^2	newton/meter^2	1.00 E − 1*
foot of water (39.2 °F)	newton/meter^2	2.988 98 E + 3
inch of mercury (32 °F)	newton/meter^2	3.386 389 E + 3
inch of mercury (60 °F)	newton/meter^2	3.376 85 E + 3
inch of water (39.2 °F)	newton/meter^2	2.490 82 E + 2
inch of water (60 °F)	newton/meter^2	2.4884 E + 2
kgf/centimeter^2	newton/meter^2	9.806 65 E + 4*
kgf/meter^2	newton/meter^2	9.806 65 E + 0*
lbf/foot^2	newton/meter^2	4.788 025 8 E + 1
lbf/inch^2 (psi)	newton/meter^2	6.894 757 2 E + 3
millibar	newton/meter^2	1.00 E + 2*
millimeter of mercury (0 °C)	newton/meter^2	1.333 224 E + 2
pascal	newton/meter^2	1.00 E + 0*
psi (lbf/inch^2)	newton/meter^2	6.894 757 2 E + 3
torr (0 °C)	newton/meter^2	1.333 22 E + 2
Speed		
foot/second	meter/second	3.048 E − 1*
inch/second	meter/second	2.54 E − 2*
kilometer/hour	meter/second	2.777 777 8 E − 1
knot (international)	meter/second	5.144 444 444 E − 1
mile/hour (U.S. statute)	meter/second	4.4704 E − 1*

Table E.1 (continued)

To convert from	to	Multiply by
Temperature		
Celsius	kelvin	$t_K = t_C + 273.15$
Fahrenheit	kelvin	$t_K = (5/9)(t_F + 459.67)$
Fahrenheit	Celsius	$t_C = (5/9)(t_F - 32)$
Rankine	kelvin	$t_K = (5/9)t_R$
Time		
day (mean solar)	second (mean solar)	8.64 E + 4*
hour (mean solar)	second (mean solar)	3.60 E + 3*
minute (mean solar)	second (mean solar)	6.00 E + 1*
year (calendar)	second (mean solar)	3.1536 E + 7*
Viscosity		
centistoke	meter^2/second	1.00 E − 6*
stoke	meter^2/second	1.00 E − 4*
foot^2/second	meter^2/second	9.290 304 E − 2*
centipoise	newton second/meter^2	1.00 E − 3*
lbm/foot second	newton second/meter^2	1.488 163 9 E + 0
lbf second/foot^2	newton second/meter^2	4.788 025 8 E + 1
poise	newton second/meter^2	1.00 E − 1*
poundal second/foot^2	newton second/meter^2	1.488 163 9 E + 0
slug/foot second	newton second/meter^2	4.788 025 8 E + 1
rhe	meter^2/newton second	1.00 E + 1*
Volume		
acre foot	meter^3	1.233 481 837 547 52 E + 3*
barrel (petroleum, 42 gallons)	meter^3	1.589 873 E − 1
board foot	meter^3	2.359 737 216 E − 3*
bushel (U.S.)	meter^3	3.523 907 016 688 E − 2*
cord	meter^3	3.624 556 3 E + 0
cup	meter^3	2.365 882 365 E − 4*
dram (U.S. fluid)	meter^3	3.696 691 195 312 5 E − 6*
fluid ounce (U.S.)	meter^3	2.957 352 956 25 E − 5*
foot^3	meter^3	2.831 684 659 2 E − 2*
gallon (U.K. liquid)	meter^3	4.546 087 E − 3
gallon (U.S. liquid)	meter^3	3.785 411 784 E − 3*
inch^3	meter^3	1.638 706 4 E − 5*
liter	meter^3	1.00 E − 3*
ounce (U.S. fluid)	meter^3	2.957 352 956 25 E − 5*
peck (U.S.)	meter^3	8.809 767 541 72 E − 3*
pint (U.S. liquid)	meter^3	4.731 764 73 E − 4*
quart (U.S. liquid)	meter^3	9.463 529 5 E − 4
stere	meter^3	1.00 E + 0*
tablespoon	meter^3	1.478 676 478 125 E − 5*
teaspoon	meter^3	4.928 921 593 75 E − 6*
yard^3	meter^3	7.645 548 579 84 E − 1*

Boundary Layer Theory. Lift and Drag.

$$p + \frac{\rho U^2}{2} = const$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$D = \rho b U^2 \theta$$

$$dD = b\tau_W dx$$

$$C_L = \frac{L}{1/2\,\rho U^2 A}$$

$$C_D = \frac{D}{1/2\,\rho U^2 A}$$

$$C_f = \frac{\tau_w}{1/2\,\rho U^2}$$

$$S = \frac{nD}{U}$$

$$\tau_W = 0.332 U^{3/2}\sqrt{\frac{\rho\mu}{x}}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}};\ \frac{\delta^*}{x} = \frac{1.721}{\sqrt{Re_x}};\ \frac{\theta}{x} = \frac{0.664}{\sqrt{Re_x}}$$

Gas Dynamics

$$\dot{m} = \rho V A$$

$$\rho = \frac{p}{RT}$$

$$R = \frac{\lambda}{M_W}$$

$$C_p - C_V = R$$

$$k = \frac{C_p}{C_V}$$

$$\Delta U = C_V \Delta T$$

$$\Delta H = C_p \Delta T$$

$$\Delta S = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$\Delta S = C_V \ln\frac{T_2}{T_1} + R\ln\frac{\rho_1}{\rho_2}$$

$$\left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{\rho_2}{\rho_1}\right)^k = \frac{p_2}{p_1}$$

$$c = \sqrt{RTk}$$

Isentropic Flow through a Nozzle

$$\frac{T}{T_0} = \frac{1}{1 + [(k-1)/2]Ma^2}$$

$$\frac{p}{p_0} = \left\{\frac{1}{1 + [(k-1)/2]Ma^2}\right\}^{k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left\{\frac{1}{1 + [(k-1)/2]Ma^2}\right\}^{1/(k-1)}$$

$$\frac{A}{A^*} = \frac{1}{Ma}\left\{\frac{1 + [(k-1)/2]Ma^2}{1 + [(k-1)/2]}\right\}^{\frac{k+1}{2(k-1)}}$$

Fanno Flow

$$\frac{T}{T^*} = \frac{(k+1)/2}{1 + [(k-1)/2]Ma^2}$$

$$\frac{V}{V^*} = \left\{\frac{[(k+1)/2]Ma^2}{1 + [(k-1)/2]Ma^2}\right\}^{1/2}$$

$$\frac{p}{p^*} = \frac{1}{Ma}\left\{\frac{(k+1)/2}{1 + [(k-1)/2]Ma^2}\right\}^{\frac{1}{2}}$$

$$\frac{p_0}{p_0^*} = \frac{1}{Ma}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}Ma^2\right)\right]^{\frac{k+1}{2(k-1)}}$$

$$\frac{1}{k}\frac{(1 - Ma^2)}{Ma^2} + \frac{k+1}{2k} ln\left\{\frac{[(k+1)/2]Ma^2}{1 + [(k-1)/2]Ma^2}\right\} = \frac{f(l^* - l)}{D}$$

Rayleigh Flow

$$\frac{p}{p_a} = \frac{1+k}{1 + kMa^2}$$

$$\frac{V}{V_a} = \frac{\rho_a}{\rho} = Ma\left[\frac{(1+k)Ma}{1 + kMa^2}\right]$$

$$\frac{T}{T_a} = \left[\frac{(1+k)Ma}{1 + kMa^2}\right]^2$$

$$\frac{T_0}{T_{0,a}} = \frac{2(k+1)Ma^2\left(1 + \frac{k-1}{2}Ma^2\right)}{(1 + kMa^2)^2}$$

$$\frac{p_0}{p_{0,a}} = \frac{1+k}{1 + kMa^2}\left[\left(\frac{2}{1+k}\right)\left(1 + \frac{k-1}{2}Ma^2\right)\right]^{k/(k-1)}$$

Normal shock

$$Ma_y^2 = \frac{Ma_x^2 + [2/(k-1)]}{[2k/(k-1)]Ma_x^2 - 1}$$

$$\frac{\rho_y}{\rho_x} = \frac{V_x}{V_y} = \frac{(k+1)Ma_x^2}{(k-1)Ma_x^2 + 2}$$

$$\frac{p_y}{p_x} = \frac{2k}{k+1}Ma_x^2 - \frac{k-1}{k+1}$$

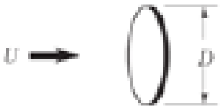
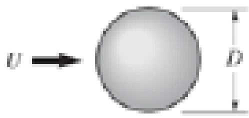
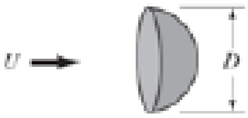
$$\frac{T_y}{T_x} = \frac{\{1 + [(k-1)/2]Ma_x^2\}\{[2k/(k-1)]Ma_x^2 - 1\}}{\{(k+1)^2/[2(k-1)]\}Ma_x^2}$$

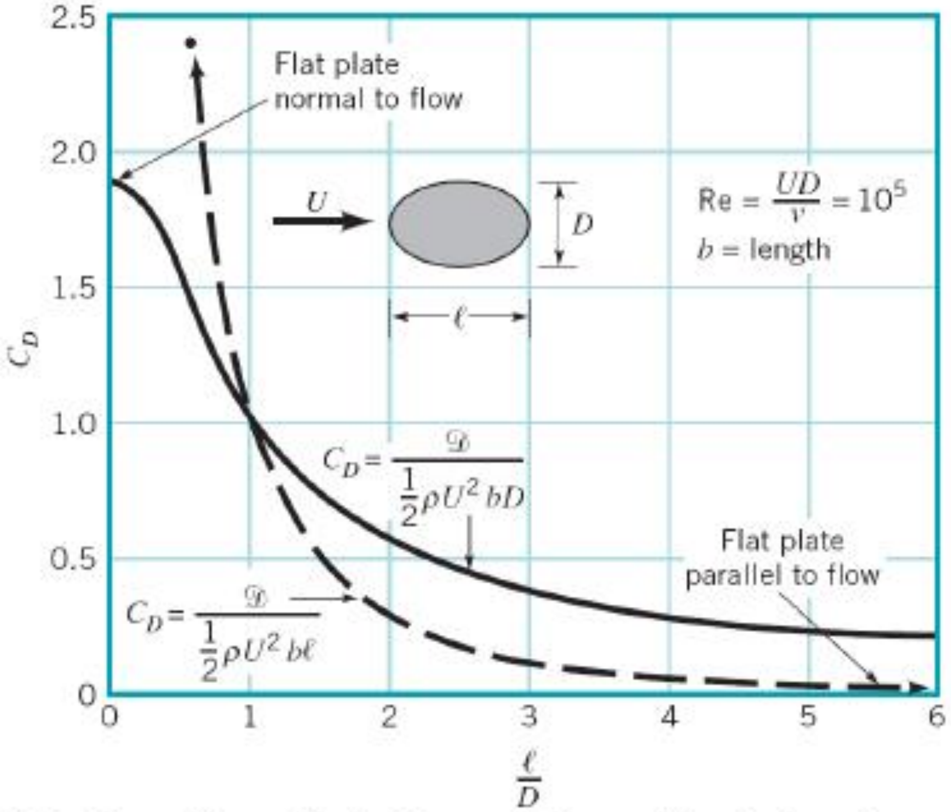
$$\frac{p_{0,y}}{p_{0,x}} = \frac{\{[(k+1)/2]Ma_x^2\}^{k/(k-1)}\{1 + [(k-1)/2]Ma_x^2\}^{k/(1-k)}}{\{[2k/(k+1)]Ma_x^2 - (k-1)/(k+1)\}^{1/(k-1)}}$$

Empirical Equations for the Flat Plate Drag Coefficient
Table 9.3 in Fundamentals of Fluid Mechanics, Munson et al, 6th edition, p.487

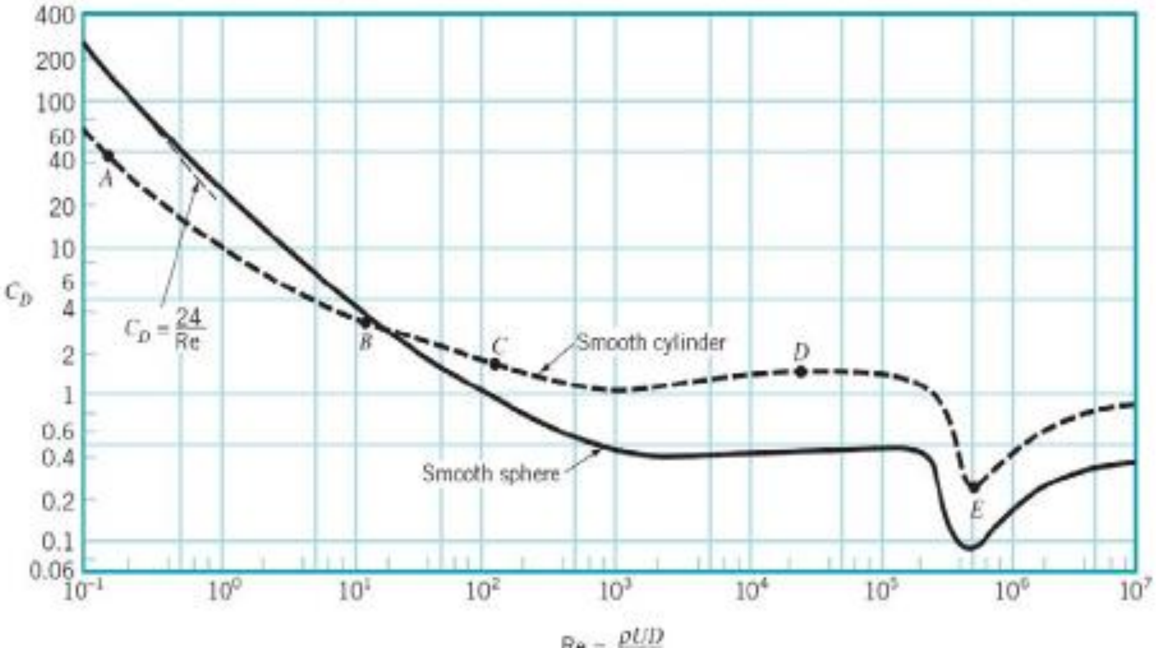
Equation	**Flow Conditions**
$C_{Df} = 1.328/(\mathrm{Re}_\ell)^{0.5}$	Laminar flow
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58} - 1700/\mathrm{Re}_\ell$	Transitional with $\mathrm{Re}_{xcr} = 5 \times 10^5$
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58}$	Turbulent, smooth plate
$C_{Df} = [1.89 - 1.62 \log(\varepsilon/\ell)]^{-2.5}$	Completely turbulent

Low Reynolds Number Drag Coefficients
Table 9.4 in Fundamentals of Fluid Mechanics, Munson et al, p.499

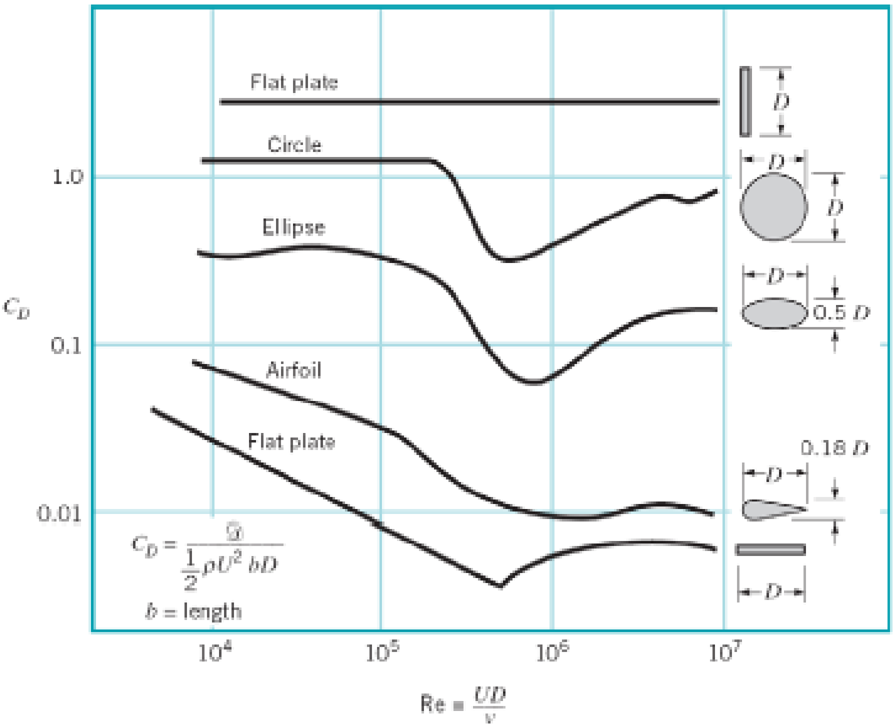
Object	$C_D = \mathscr{D}/(\rho U^2 A/2)$ **(for Re ≲ 1)**	**Object**	C_D
a. Circular disk normal to flow 	20.4/Re	c. Sphere 	24.0/Re
b. Circular disk parallel to flow 	13.6/Re	d. Hemisphere 	22.2/Re



Drag coefficient for an ellipse with the characteristic area either the frontal area, $A=bD$, or the planform area, $A=bl$.
Figure 9.19 in Fundamentals of Fluid Mechanics, Munson et al, p.498.



Drag coefficient as a function of Reynolds number for a smooth cylinder and a smooth sphere.
Figure 9.21 in Fundamentals of Fluid Mechanics, Munson et al, p.501.



Character of the drag coefficient as a function of Reynolds number for objects with various degrees of streamlining.
Figure 9.22 in Fundamentals of Fluid Mechanics, Munson et al, p.502.